**Markets and Information**

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In markets, individuals’ expectations about payoffs matter for how they will choose.

Unknown desirability of different alternatives is either exogenous or endogenous. Exogenous desirability means that an alternative is in essence good or bad idea regardless the decisions of individuals. Endogenous desirability depends on actual decisions of people.

**Markets with Exogenous Events**

The events are exogenous if the probabilities of the events are not affected by the outcomes of the market. A very classical example is prediction markets. For example, forecasting the election results, stock prices or sporting.

**Horse Races, Betting, and Beliefs**

Suppose there are two horses A and B, on of which will win the race. We are assuming that the bettor plans to bet all his money w on the two horses, r (fraction) to bet on A and 1-r to bet on B. Suppose that horse A will win with probability a, then B will win with probability 1-a (no tie is assumed). For the odds, oA means that when investing x dollars on horse A and if it wins, that will return oAx dollars to the bettor. Odds OB represents the odds for horse B.

* **Modeling risk and evaluating the utility of wealth**

We proceed with the assumption that people would not choose the strategy where there is plausible scenario where their savings would become 0. Each player evaluates each strategy in accordance with the expected value of it payoffs. In this regard, we need a numerical way to specify the evaluation of the bettor’s wealth. The utility function U(.) would help us do this.

Linear utility function: U(w) = w or U(w)=aw+b, the rate of increase remains stable.

U(w) = w1/2 or U(w) =ln(w), the rate of increase slows down.

* **Logarithmic utility**

Suppose that the bettor’s utility function is the natural logarithmic of wealth, ln(w). The rate of growth slows down with increasing wealth. The increase in the utility is the same when you are doubling your wealth:

Ln(2w) – ln(w) = ln(2)

* **Optimal strategy on horse race**

The probability of horse A to win is a, horse B to win is 1-a.

Odds for horse A is oA and odds for horse B is oB.

Suppose that the bettor invests fraction r of his wealth w on A and 1-r on B. What’s the optimal fraction of r if the utility function is ln(w)?

Expected utility = a\*ln(oArw) + (1-a)\*ln(oBw(1-r)) = constant + a\*ln(r) + (1-a)ln(1-r)

Take derivatives according to r

a/r – (1-a)/(1-r) = 0

r= a is the optimal solution

**Aggregate beliefs and the “wisdom of crowds”**

When there are n bettors, we have to aggregate their beliefs towards the race. For individual 1, 2, 3, …n, suppose that each individual believes that horse A will win with probability ai, horse B will win with probability bi = 1-ai. Wealth of individual i is wi. In this regard, total wealth

w = w1+w2+…+wn

investment on horse A

a1w1+a2w2+ … +anwn

the return if horse A wins: a1w1oA + a2w2oA + … +anwnoA

in order for the amount paid to bettors if horse A win is equal to the money that track collects

a1w1oA + a2w2oA + … +anwnoA = w

oA-1 = a1w1/w + a2w2/w + … anwn/w = a1f1 + a2f2 + …anfn

similarly, oB-1 = = b1f1 + b2f2 + …bnfn

oA-1 + oB-1 = 1

**State prices in stock market**

Company 1 is worth 2 dollars in state 1 s1 and 1 dollar in state 2 s2; while company 2 is worth 1 dollar in state 1 and 2 dollars in state 1. Use v1 to represent prices for stock in company 1 and v2 prices for stock in company 2.

P1 nad p2 denote state prices for state s1 and s2.

V1= 2p1 +p2

V2 =p1+2p2

**Markets with Endogenous Events**

When the individuals’ beliefs about an event influence their actions and those actions have an effect on the realization of that event, this event is defined as *endogenous* (e.g. bankruptcy, used-car market, party). In a way, they are self-fulfilling expectations. When the buyer or the seller has an information that the other does not know, this is called *asymmetric information* (e.g. second-hand market, health insurance).

**The Market for Lemons**

Assumptions:

1. The items have varying qualities.
2. Asymmetric information about quality (seller > buyer) but the buyer is aware of this.
3. There is a different reservation price for each quality level and pbuyer ≥ pseller.
4. All items (high or low quality) are sold at a uniform price (p) and the seller offers an item if p ≥ pseller.
5. There are more buyers than the items for sale.

If we had *symmetric information*, then all items would be up for sale and the sales price p would be between pbuyer and pseller for each quality level (because of assumption (v), we will have p = pbuyer for each quality level).

In case of *asymmetric information*,

* if the proportion of high quality cars, g ≥ (pseller,high – pbuyer,low)/(pbuyer,high – pbuyer,low), then either all items will be up for sale or only low quality cars will be sold,
* otherwise, only low quality cars will be sold.

If there are high quality, low quality and lemon cars, we will have complete market failure at the equilibrium.

Asymmetric Information in Other Markets:

* Labor market (buyer=employer < seller=worker) – less productive workers (security)
* Insurance (buyer=company < seller=individual) – less healthy individuals (fear, moral hazard)
* Trading and Stock market (motivation)

**Dealing with Information Asymmetry**

In order to prevent complete market failure, we need to give incentives to increase the proportion of high quality items through market regulations (lemon law, quality regulations), repeat purchases and mitigating asymmetric information (reviews and ratings). The seller could also use some manipulations (diamond market, credence goods) to create a market.

*Signaling Quality*

Methods to alleviate information asymmetry are certification mechanism or warranty - a signal about quality and increases the proportion of high quality cars offered for sale and the sales price. In case of labor market, education could be a quality signal.

*Reputation Systems*

Buyer and seller most likely meet only once (eBay, Couchsurfing, Airbnb). An evaluation of the transaction provides a signal for quality and will increase the likelihood of good transactions.

*The challenge* is to construct a reputation system where “it is cheaper for good sellers to acquire a good reputation than it is bad for sellers to acquire the same reputation” (no multiple identities and fake reviews).

*Ad Quality in Keyword-Based Advertising*

The trade-off is between the short-term gains from high-clickthrough low quality ads and the long-term losses due to user perception of quality. The strategy should be a mix of organic search and paid search; otherwise, the users will be disappointed with the relevancy of the content. The desired equilibrium contains users with high expectations of high quality ads and advertisers with high-quality content. SEO and SEM are tools to improve the equilibrium so that the search engine delivers high quality content to the users.

**Wealth Dynamics in Markets**

The intuition about market evolution over time:

1. “Natural selection” – those who make better decisions tend to stay in the game more, acquire more wealth and have a greater weight on the outcome of the game (more rational investors and more efficient stock markets)
2. The market acts as an artificially intelligent Bayesian agent that aggregates information. The predicted probability of each hypothesis is updated after an observation according to Bayes’ rule.

*Convergence with the correct hypothesis:*

Horse race between A & B

(a,b) – actual probabilities of winning (a = 1-b)

(an,bn) – hypothesis n for probabilities, fn =Pr(an,bn)– prior prob. on hypothesis n, n=1,…,N

Let hypothesis 1 be the correct hypothesis w.l.o.g. (a1 = a, b1 =b)

S – observed sequence of outcomes, T - # of trials, k - # of wins for A, l - # of wins for B

As the number of observations goes to infinity, the predicted probability (posterior probability, updated with Bayes’ rule) of the correct hypothesis goes to 1 as the probabilities of all the others go to zero.

*Convergence without the correct hypothesis:*

Define the distance between a hypothesis and the truth (relative entropy)

Let the hypothesis 1 be better than all the others w.l.o.g. Then

As the number of observations goes to infinity, the predicted probability (posterior probability, updated with Bayes’ rule) of the best hypothesis goes to 1 as the probabilities of all the others go to zero.

*Evolution of Wealth Shares*

Horse race between A & B

(an,bn) – belief of bettor n on probabilities, wn – initial wealth of bettor n ( = w), fn =wn/w,

–wealth of bettor n at time t, n=1,…,N

and – odds on winning at time t (depends on who the bettors are and how much they bet)

Bets on time t : an on A and bn on B

Bets on time t +1: an on A and bn on B

S – observed sequence of outcomes, k - # of wins for A, l - # of wins for B

The ratio of wealth shares between bettor m and n is .

The inverse odds follow the results of Bayesian learning because they are computed from the wealth-share-weighted average of the bettors’ beliefs.

Wealth shares of the bettors behave exactly like the posterior probabilities on hypotheses. The wealth is adjusted in favor of a better bettor and in the limit, the best bettor acquires all wealth.

*Extensions*

* The bettors’ beliefs could also be changing over time.
* The bettors may consider other investment options.

*Interpretations*

* The market behaves according to the best trader. The crowd is as smart as the smartest in the limit.
* If there are not enough traded assets, the worse traders may not be driven out of the market.